AD-A172 110 COMPARATIVE ACCURACY OF FIVE INDICES OF DIMENSIONALITY OF BINARY ITEMS(U) ILLINOIS UNIV AT URBANA DEPT OF PSYCHOLOGY L R TUCKER ET AL. JAN 86 N00014-84-K-0186 1/1

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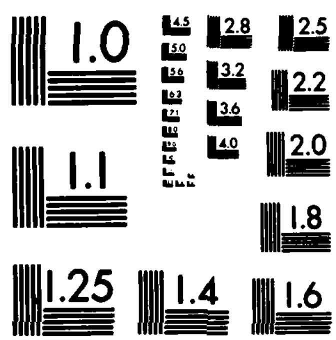
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COMPARATIVE ACCURACY OF FIVE INDICES OF DIMENSIONALITY OF BINARY ITEMS

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January 1986

This research was sponsored by

Personnel and Training Research Programs
Psychological Sciences Division
Office of Naval Research

Under Contract No. N00014-84-K-0186
Contract Authority No. NR 150-533/4-3-85

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AD-A172 110

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b.
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Department of Psychology University of Illinois 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Personnel and Training Research Programs Office of Naval Research (Code 1142PT)
6c. ADDRESS (City, State, and ZIP Code) 603 East Daniel Street Champaign, IL 61820	7b. ADDRESS (City, State, and ZIP Code) 800 North Quincy Street Arlington, VA 22217-5000
8a. NAME OF FUNDING / SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-84-K-0186
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61153N PROJECT NO. RR04204 TASK NO. RR04204-01 WORK UNIT ACCESSION NO. NR 150-533

11. TITLE (Include Security Classification)
Comparative Accuracy Of Five Indices Of Dimensionality Of Binary Items

12. PERSONAL AUTHOR(S) Tucker, Ledyard R; Humphreys, Lloyd G.; Roznowski, Mary A.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical Research	FROM ______ TO ______	January 1986	44

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
05	09		dimensionality, binary items, factor analysis

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A Monte Carlo study of five indices of dimensionality of binary items has been conducted using a computer model that allowed sampling of both items and people. Five parameters were systematically varied in a factorial design: (1) number of common factors from one to five; (2) number of items, including 20, 30, 40, and 60; (3) sample sizes of 125 and 500; (4) nearly rectangular and highly peaked distributions of item difficulties; and (5) factor intercorrelations averaging in the thirties and in the fifties. Accuracy in distinguishing one factor from more than one was the criterion. An index involving variance-covariance matrices and based on the unidimensional property of local independence of items was overall most accurate. This index and an index based on ratios of differences in successive Eigenvalues were substantially more accurate in the peaked than in the rectangular distributions of item difficulties. An index based on the pattern of second principal factor loadings obtained in product-moment correlation matrices showed the reverse pattern. For all indices there is an increase in accuracy as both sample size and number of items is increased; for all indices

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Susan Chipman	22b. TELEPHONE (Include Area Code) 202-696-4318 22c. OFFICE SYMBOL ONR 1142PT

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

19. Abstract (continued)

there is a decrease in accuracy as factor intercorrelations are increased and as the number of factors is increased. Although the effect on accuracy of the index as a function of the last four parameters named is in the same direction, the indices are differentially sensitive to the levels of treatment. The interaction between parameter and index, however, is largest for the distribution of item difficulties.

COMPARATIVE ACCURACY OF FIVE INDICES OF DIMENSIONALITY OF BINARY ITEMS

Introduction

The standard method of testing the dimensionality of binary items is to compute tetrachoric intercorrelations, obtain the principal components, and inspect the latent roots. If the items are unidimensional, one expects the first root to describe a major although numerically uncertain proportion of the total variance. One also expects a large difference in the size of the second root relative to the first and no sizable gap subsequent to the first difference. These evaluations, however, have no firm quantitative basis.

Tetrachoric correlations are used for the component analysis because the size of product-moment correlations is affected by the difficulties of the items. Everyone knows that these correlations (phi coefficients) produce difficulty level factors. If items were to produce a perfect Guttman scale, the matrix of intercorrelations would be a perfect simplex, not a Spearman hierarchy.

Even though the use of tetrachorics is more or less standard, the R-matrix has two undesirable properties for purposes of factor analysis and the determination of dimensionality. The matrix is very likely to be non-Gramian, and the sampling errors of the individual tetrachorics vary widely as a function of the difficulty levels of the items correlated. The effects of these properties become more severe as the spread of item difficulties increases.

Importance of Dimensionality. The problem of dimensionality has come to the fore with the increase of interest in Item Response Theory and the use of that methodology in adaptive testing. Although dependable item and

person parameters can be obtained from item pools that are not unidimensional in the strict sense of that term--a dominant dimension defined by correlated group factors is sufficient (Drasgow and Parsons, 1983) and more valid in most applications (Humphreys, 1985)--a method of test administration that potentially provides a different set of items for each examinee and regularly provides a different set of items for subgroups of examinees places a greater demand on the IRT assumption of unidimensionality.

It becomes important, therefore, to determine the accuracy with which a decision contrasting unidimensionality with multidimensionality can be made from the matrix of tetrachoric correlations. It is also important to evaluate nontraditional ways of evaluating dimensionality. In order to do this it is essential to have a model of known dimensionality, to sample both items and examinees, and to compare possible indices of dimensionality in the same data sets.

Characteristics of a Desirable Model

Our objective was to develop a model that could simulate realistically psychological data, especially cognitive data. The starting point was the model developed by Tucker, Koopman, and Linn (1969). Those authors discussed both a "formal" factor model and a "simulation" model. The models differ in that the latter adds a relatively small amount of variance from a large number of small, overlapping factors. The added variance simulates the large number of determinants of responses to test items. Neither items nor tests are ever pure measures of a hypothetical factor. We plan to investigate both models as they affect detection of multidimensionality, but we shall initially report results for the formal model only.

A second requirement was that the model should furnish data realistic in its psychometric properties. Distributions of item difficulties should

approximate those that are encountered in practice. Item correlations should be at levels encountered in most item pools. Product-moment correlations among cognitive items in wide ranges of talent are rarely greater than .25 and are frequently considerably lower. Differences in the size of mean correlations between a rather heterogeneous intelligence test and a homogeneous test of mechanical information are small. Tetrachoric correlations are larger, but generally well below the maximum value of 1.00 expected in a perfect Guttman scale. Tetrachoric correlations of unity do occur in samples of reasonable size but only when a very easy item is correlated with a very difficult item. A zero frequency in one cell of the four-fold table can readily occur by chance in such combinations.

A third requirement is that the model should be a compensatory one. In this initial study, an item's variance is broken down into a linear combination of loadings on major group factors and random error. Intercorrelations are the sums of products of overlapping factors. A compensatory model seems to fit psychological data rather well, including the predictive validities of psychological tests for practical criteria.

Description of the Model

The data in this study were generated in such a way that the structural relations desired were known and prespecified. This was desired so that the validity of the factor analytic design could be checked. With such a generation method, a design plan can be adjusted by the experimenter in terms of the exact nature and quality of the data.

Basic Notions. The model used here specifies the existence of a major domain, which represents the most important influences on individuals' observed

scores. Consider factor matrix B whose entries are the actual input loadings. Every row (i) in B represents a variable, while every column (j) in B represents a factor. Each element in B (B_{ij}) is an input factor loading. These elements are the conceptual loadings or values in the major domain that represent ideas or theoretical notions concerning the makeup and structural relations of the input variables under study.

The major domain was presumed to have simple structure. Further, a restriction of the factor matrix to positive manifold was desired. This latter characteristic was required because the data being simulated were cognitive ability items.

Factor Intercorrelations. An initial step in the process of generating the factor model was determining the phi matrix (Ø, intercorrelations among factors). Significant control of this parameter was available to the experimenter.

A value is set by the experimenter in the range 0,1 that will indirectly determine a minimum level of the factor correlations. A second, higher value in the same range is then chosen from which the first is subtracted. The difference is multiplied by a random number in the interval 0,1 and the product added to the initial value. The procedure is repeated for each factor in the model using, for the present, the same first and second values. The result is a unique value for each factor.

The next step determines the correlation between pairs of factors. This involves taking the square root of the values determined for each factor in the preceding step and obtaining the cross-products among all factors. In effect, the factor intercorrelations are the products of factor loadings on a single general factor.

The factor correlations used in the present application of the model were determined by initial values of .20 and .40 for the low and .50 and .70 for the high levels, respectively. Thus the random increments to the base were, on average, the same size for the two levels of factor correlations.

Ensuring Simple Structure. The next step in the generation of the major domain is deriving the first order $\tilde{B}$ having simple structure with a restriction to positive manifold. Values for the intial $\tilde{B}$ are chosen by selecting a random value in the interval 0,1.

Several checks are carried out at this point to assure that the structural relations and the general quality of the factor matrix are known. These are listed here:

1. A cut off probability for establishing zeros in $\tilde{B}$ is used at this point to determine the actual number of zeros in each column of $\tilde{B}$ (i.e., per factor). In effect, this step establishes the simple structure aspect of the factor matrix.

2. A second precaution is carried out to insure the presence of at least one high (nonzero) entry in each row (i.e., per variable) of the factor matrix. If no nonzero loading is present, one is inserted in one of the columns. The position of the nonzero entry is chosen randomly.

3. The third check concerns a count of the high loadings in each column of the factor matrix (i.e., for each factor). Again, the investigator determines *a priori* the minimum number of high loadings desired. As before, high entries are inserted until the minimum number of high loadings desired is achieved.

4. Lastly, all pairs of columns (j and k, $j \neq k$) are compared to determine whether there are the appropriate (specified *a priori*) number of rows with a high entry on j and zero entry on k, again a further assurance

that each variable is represented and that simple structure is achieved. Adjustments are made when necessary.

Size of Factor Loadings. A final set of adjustments is carried out to effect the actual size of the loadings in the factor matrix (loadings are either Ø.Ø or .5 before this adjustment). The investigator again has input into the absolute size of factor loadings. The investigator determines two initial values which indicate the range from which loadings for the nonzero entries in the factor matrix are chosen. This process is the same as that used for the generation of the Ø matrix. The procedure presented earlier for second order Gl coefficients is used at this point also. A vector of such coefficients is obtained, one coefficient for each variable. These coefficients are random rectilinearly distributed from Gl_S to Gl_E. (Gl_S and Gl_E are defined as above and are selected a priori by the investigator). The Gl coefficients are the hypothesized variance from the major conceptualized domain of influences.

At this point, the matrix A, which is the factor weight matrix transformed to uncorrelated factors, is obtained. The first step in this process is computing the Cholesky decomposition ($\underline{v}$) of the matrix phi (Ø) determined as above. Matrix B is post multiplied by $\underline{v}$ to obtain an initial A matrix. Next, rows of A and B are adjusted to take into account the vector of computed Gl coefficients (again, the desired variance attributable to the major domain of influences). After rows of A are normalized, the same multiplier is applied to B to rescale B in accordance with the transformation to A.

Application to Binary Scores. Up to this point the model has been described in terms of continuous variates. The model in this section is converted to binary measures. (Each individual has a measure (score) on each

of the k dimensions.) Definitions for scores on dimensions and their components are as follows:

X_{ik} = score or measure of individual i on dimension k.

Z_{ij} = item score of individual i on item j.

W_{jk} = weight for item j on dimension k.

A fundamental relation is:

(1) $Z_{ij} = \sum_k X_{ik} W_{jk}$

To generalize this relation, we can see that:

(2) $\underline{Z}_i = \underline{X}_i \Omega'$

where: $\underline{X}_i$ is a vector of scores X_{ik}, and $\underline{Z}_i$ is a vector of scores Z_{ij}. Ω = matrix of weights W_{jk}.

A further look at the Ω (weight) matrix reveals "common" factor weights, "specific" factor weights, and "error of measurement" factor weights. Specific and error factors are also considered together as uniqueness component or U. We designate the common factor weight matrix A as before.

The weight matrix Ω can be decomposed from a super matrix to two separate weight matrices, A and U representing weights from common influences and unique influences, respectively. For the purposes of this study, the vector $\underline{X}_i$ can be similarly partitioned into common and unique components $\underline{a}_i$, $\underline{u}_i$, respectively, $\underline{X}_i = \langle \underline{a}_i, \underline{u}_i \rangle$. Expanding the above relation (2), we see:

(3) $Z_{ij} = \underline{a}_i A + \underline{u}_j U$

With this basic factor model as a start, individual binary data (scores) are derived that conform to the qualities of the simulation model in continuous form. The factor model is a linear model involving the addition of the contributions to item scores. The factor weight matrix A (common factor influences) is applied to the underlying score distribution and combined with the unique components to determine item scores. To compute Z_{ij} (item score of individual i on item j)

from (1), a random normal deviate for individual i on each dimension k (X_{ik}) is chosen and multiplied by factor loading from A for item j on dimension k (a_{jk}).

For a three-factor model, the following holds:

(4) $Z_{ij} = X_{i1}a_{j1} + X_{i2}a_{j2} + X_{i3}a_{j3}$

To the score Z_{ij} is added the uniqueness component. Again, a random normal deviate is chosen (one for each item) which is multiplied by the uniqueness for that item. This product is added to the Z_{ij} from the common factors.

<u>Generation of Binary Responses</u>. To generate item binary scores, an individual's normally distributed continuous score for item j (Z_{ij}) is compared to a cutting score or threshold for item j. The cutting score distribution is derived with input from the investigator. The cutting score mean and standard deviation are determined <u>a priori</u> and are used to set up the cutting score values (d_j), one for each item j. Further, a vector of guessing parameters (c_j) is set up taking into account a range of guessing probabilities specified by the investigator. The equations that characterize the probabitistic model used for generating binary responses follow. S_{ij} represent the individual's binary response to item j. S_{ij} is initially set to 0.

5a) IF $(Z_{ij} \geq d_j)$ $S_{ij} = 1$;

5b) IF $(Z_{ij} < d_j)$ $P(S_{ij} = 1) = c_j$.

The event of S_{ij} equalling 1 or 0 is independent from occasion to occasion. Thus, if the score of individual i to item j is greater than, or equal to, the cutting score for item j, the individual has correctly answered the item. If the score (Z_{ij}) is less than the threshold, the individual receives a zero to indicate an incorrect response. However, a guessing component is taken into account such that the individual's wrong answer to item j is changed to a correct answer with probability c_j. This process is continued for all j items of the i individuals. The model can be seen as a probabilistic model that involves each measure (item) as a sampling of influences on the individual's behavior.

Independent Parameters

Five parameters were varied in a factorial design. The variations defined a range of possible values for each parameter, but no attempt was made at this stage to fill in the gaps. Cells in the design were filled with 20 independent samples with the exception of those defined by the largest number of items. Only 10 replications were obtained in the latter case to conserve computer time.

Sample Sizes. These were set at 125 and 500, providing a 2 to 1 ratio of expected sampling errors. Because this parameter is under the control of the research person, an adequate sample size (N) can usually be established in advance. All indices of dimensionality should vary in accuracy as a function of this parameter.

Number of Items. This parameter (n) was varied as follows: 20, 30, 40, and 60. Number of items can also be directly controlled by the research person. For a given number of factors one expects to be able to define factors more accurately the larger the number of items.

Item Difficulties. Two distributions were used, one relatively flat, the other peaked. These were defined in normal deviate units as follows: mu = .10, σ = .80; mu = -.13, σ = .32. The positive sign of the mean indicates a central tendency of p-values less than .50 in the absence of guessing. The mean p-values for both are greater than .50 after the guessing parameter has been applied. An experienced test constructor can exercise an approximate degree of control over this parameter. More importantly, item difficulties are always knowable before a decision concerning dimensionality has to be made. Note that variation in the distribution of item difficulties affects the size of the item product-moment intercorrelations. It affects only the sampling variability of tetrachorics.

Factor Intercorrelations. The manipulation of this parameter produced mean correlations of approximately .35 and .55. This difference in size did, of course, produce variation in size of item intercorrelations of both types. For purposes of determining dimensionality factor intercorrelations are not knowable, but one can obtain the mean item correlation in advance of a decision. For a given set of items the mean product-moment correlation is readily computed from an internal consistency statistic for the total score.

Number of Factors. This parameter was varied from 1 through 5. A test constructor can only control it approximately by adherence to item specifications. However, good test construction practices will not increase the number of major factors as the number of items is increased. On the other hand, the size of the correlations among the multiple factors that remain, when unidimensionality is not achieved by the item writer who is working toward that goal, is likely to be high. If unidimensionality can be rejected, it is also important to decide on the number of group factors, but this decision is not central to the present research. It should also be noted that variation of this parameter in our model results in some degree of variation of the level of item intercorrelations. In this case there is variation in both product-moment and tetrachoric correlations.

Given the oblique factors that are characteristic of cognitive data, one expects increasing difficulty in distinguishing between unidimensionality and multidimensionality as the number of factors increases. Positively correlated first-order factors determine one or more second-order factors, but in our present model there is only one second-order factor. It describes an increasing amount of total variance as the number of factors increases. When the general factor is held constant, the total contribution of five factors is less than the total contribution of two factors In an important sense, as the number

of homogenously correlated group factors increases, the more nearly do the data appear to be unidimensional and the more difficult the task becomes in both our model and real data.

Dependent Variables

In addition to the use of indices based on nontraditional approaches to dimensionality using covariances and product-moment correlations, we also used indices based on the Eigenvalues of R-matrices composed of tetrachoric correlations. The principal factors model, substituting the highest correlation in a column for the unity of the R-matrix, was used for these indices in place of the principal components. This decision was based on preliminary research in which the differences between the two models were trivial in size, but on average favored the principal factors.

Eigenvalue Differences. The size of the difference between the first two Eigenvalues may represent most closely the basis for a decision concerning dimensionality obtained by "root staring", i.e., by inspection of the pattern of the latent roots. Because the size of the roots is a function of the level of item intercorrelations, the absolute size of the difference must be standardized. Each difference, therefore, was divided by the mean of the first of the two Eigenvalues. This standardized difference approximates the information that would be furnished by an index consisting of the ratio of the first two Eigenvalues for each sample in the cell.

Ratios of Differences. A ratio of the intial difference to later differences better represents the shape of the curve of the Eigenvalues. In this research we computed the ratio of the difference between the first two roots to the mean of the following two differences. If a set of items is measuring a single dimension, not only should there be a large drop in size from the first root to the second, but subsequent decreases should be small.

<u>Local Independence Criterion</u>. If unidimensionality is present, the intercorrelations of items in the population of persons who are at the same level of the latent trait are equal to zero. Local independence can be approximated in fallible data by restricting analyses to those who have the same total score on the set of items. One cannot hope to find an index of dimensionality by analyzing $\underline{n}$ different matrices independently, one for each level of total score, so the following procedure was developed:

1) An aggregate variance-covariance matrix is formed from the separate matrices computed in samples of persons having the same total score. Each sub-matrix is weighted by its sample size in forming the aggregate.

2) Signs of the aggregate covariances are changed in accordance with the sign-changing procedure of centroid factor analysis to maximize the algebraic sum of the C-matrix. Because this C-matrix held total score constant, there are approximately equal numbers of positive and negative signs in the aggregate matrix.

3) The ratio of the algebraic sum of covariances following the sign change to the absolute sum is formed. Ratios that approach unity indicate the presence of more than one factor among the original item covariances; i.e., there is structure remaining in the matrix although one factor has been removed.

4) The ratio in step 3 is then compared with the ratio of algebraic to absolute sum of the values in the covariance matrix of raw scores in which total score is not held constant. For this ratio the sign change was placed in the program so that the technique would be applicable to matrices composed of both positive and negative correlations among noncognitive items.

5) The comparison at this point in our research involves subtracting (3) from (4). Among reliable cognitive items the ratio in (4) will closely approach unity whatever the dimensionality may be. The presence of multiple factors is indicated, therefore, by small differences.

These ratios were computed with and without the diagonal entry. Results have been highly similar, but ratios from which diagonals were omitted had a slight edge in accuracy. Our results section will present data only for the latter alternative.

Pattern in Factor Loadings. The first and second principal components of the product-moment intercorrelations of items comprising a perfect Guttman scale have distinctive patterns. The loadings on the first component are all positive with the largest loadings being associated with items of moderate difficulty level and the smallest with the easiest and the most difficult items. The loadings on the second component form a curve that approximates an ogive with easy and difficult items having high loadings of opposite sign and with items of moderate difficulty having loadings close to zero. Because a perfect Guttman scale is also a perfect index of unidimensionality, it was our hypothesis that patterns of loadings in the first two principal factors of the product-moment R-matrix could be used as an index of dimensionality in fallible item data. A critical assumption in the application of this hypothesis is that there is orthogonality of item difficulties and factor content when more than one factor is present in the items.

One index made use of the signs of the second principal factor of the R-matrix of product-moment correlations.

1) The matrix is factored after replacing the unities in the diagonal with squared multiple correlations.

2) Items are ranked in order of difficulty and the preponderance of signs

of the loadings on the second principal factor in the easy and difficult halves of the items determined.

3) Each aberrant item, e.g., one with a negative sign in the easy half when most items in that half have positive signs, is given a numerical value based on the number of ranks by which it is removed from the center of the distribution of difficulties.

4) These numerical values are summed over all aberrant items. Small sums, in accordance with our hypothesis, should be associated with unidimensionality.

A second index that is sufficiently independent of the first to be given separate consideration is based on information obtained from both the first and second principal factors among the product-moment correlations.

Steps (1) and (2) are the same as for the first index.

3) Each aberrant item is given a numerical value representing the product of its first and second factor loadings.

4) These numerical values are summed without regard to sign over all aberrant items. Small sums are again associated with unidimensionality.

The rationale for this index is that items so unreliable as to have little in common with other items in the test should not influence the decision concerning dimensionality. On the other hand, items that do have something in common with at least a subset of other items and have large, aberrant second factor loadings should strongly influence the dimensionality decision.

Results

Levels of Intercorrelations. Before presenting the results from the various indices of dimensionality, it will be useful to have as background the levels of intercorrelations (phis) produced by the three parameters for which effects were expected. Typical product-moment correlations in the samples appear in Table 1.

Table 1

Representative Mean Item Correlations

As a Function of Three Parameters

	Number of Factors								
	1	2		3		4		5	
		High	Low	High	Low	High	Low	High	Low
Wide	.215	.18	.17	.16	.14	.16	.135	.16	.13
Narrow	.30	.24	.22	.23	.20	.225	.19	.225	.18

Table 2 shows the Kuder-Richardson coefficients for several selected levels of item intercorrelations that cover the range of values in Table 1.

Table 2

Kuder-Richardson Coefficients As a Function of Number of Items and Selected Values of Mean Item Correlations

Mean Item	Number of Items			
Correlations	20	30	40	60
.30	.90	.93	.94	.96
.25	.87	.91	.93	.95
.20	.83	.88	.91	.94
.15	.78	.84	.88	.91
.12	.73	.80	.84	.89

Although we tried to maintain a fairly constant level of item correlations for factors 2 through 5 by manipulating the proportion of items loaded on a single factor, we were not quite successful. The large gap, however, is between one factor and multiple factors because there were no zero loadings in the one factor model. The distribution of item difficulties has a large effect as required by the nature of the correlations. The typical item correlation changes least as a function of the amount by which we manipulated the intercorrelations of the factors in the continuous bivariate model. Even so, as will be seen later, the differences in size of the factor intercorrelations have important consequences for the accuracy of the indices.

The range of Kuder-Richardson coefficients demonstrates that our item correlations are in the appropriate ball park for most cognitive tests. Varying the number of items compensates for low levels of item correlations. Perhaps the value that is most out of line is for one factor in the narrow distribution of item difficulties. Mean item correlations of .30 in cognitive tests are rare.

<u>A Measure of Overlap</u>. The accuracy with which one can reject a particular null hypothesis depends on the overlap of two sampling distributions. In the present research we obtained sampling distributions of indices in item intercorrelations that were based on major factors varying from one through five. Because the distributions of several indices were skewed, we realized belatedly that means and standard deviations of these distributions were inadequate for determining overlap. In this initial report we obtained our measure from hand-tallied distributions. The procedure is as follows:

In the distribution of a given index for one factor we select the value that represents one factor least well. If low values of the index are expected, this is the highest value. We then count the number of values in

each of the distributions of multiple factors that were equal to or lower than our critical first factor value. Next we select in the distributions of each of the multiple factors the value that represents the presence of more than one factor least well. Again, if high values for multiple factors are expected, this is the lowest value. We count the number of one-factor values equal to or higher than the multiple factor critical value. Skewness results in asymmetry of the two measures of overlap, so we add them together to obtain a single value that can vary from zero to 40 (20 for 60 items).

Arrangement of the Tables. Tables are arranged in accordance with a standard pattern. Small sample size is in the upper half, large in the lower half. Columns are defined in the first instance by the number of items in the test and secondarily by the two levels of factor intercorrelations. One factor data, of course, do not vary along this parameter. Rows are defined in the first instance by the comparisons of one factor in turn with two, three, four, and five. A second distinction within each of these comparisons involves the two levels of item difficulties.

Standardized Difference in Eigenvalues. Overlap information for this index is presented in Table 3. It is clear that this index represents a highly unreliable method of drawing inferences concerning dimensionality. In only a small number of the most favorable combinations of parameters did we obtain zero overlap in 20 replications. Although the results would not be identical, the ratio of the first two Eigenvalues, for which the present index is an approximation, can also be considered to have little promise.

Ratio of Differences. This index, for which overlap information is summarized in Table 4, is moderately effective for the restricted distribution of item difficulties, but quite ineffective in the wide range. In contrast to the standardized differences in Eigenvalues, ratios of differences reflect

Table 3

Amount of Overlap in Each Cell

Based on the First Difference in Roots

N=125

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	39	38	38	33	35	31	30	22
	Narrow	33	27	33	29	31	28	16	22
1 vs. 3	Wide	38	38	38	29	34	31	38	22
	Narrow	32	25	35	27	33	20	22	20
1 vs. 4	Wide	37	37	35	31	36	28	38	30
	Narrow	38	29	34	31	30	20	26	20
1 vs. 5	Wide	37	38	39	36	37	32	34	30
	Narrow	31	30	35	31	32	27	30	26

N=500

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	39	37	31	12	33	27	32	0
	Narrow	20	9	17	1	22	0	6	0
1 vs. 3	Wide	38	39	34	29	34	28	34	18
	Narrow	24	14	26	9	26	8	12	8
1 vs. 4	Wide	40	40	35	30	36	32	32	28
	Narrow	29	20	28	2	33	14	20	12
1 vs. 5	Wide	40	36	33	33	39	36	32	32
	Narrow	28	25	30	17	31	13	24	8

*Error frequencies doubled.

Table 4

Amount of Overlap on Each Cell

Based on the Ratio of Successive Differences

N=125

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	36	32	35	25	38	31	0	0
	Narrow	3	0	6	6	0	0	0	0
1 vs. 3	Wide	36	39	35	25	37	36	20	0
	Narrow	14	0	6	0	0	0	0	0
1 vs. 4	Wide	39	35	39	27	40	32	38	28
	Narrow	18	6	8	6	4	0	0	0
1 vs. 5	Wide	40	33	39	35	40	34	34	36
	Narrow	16	2	22	6	9	2	0	0

N=500

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	31	25	27	6	14	0	22	4
	Narrow	0	0	0	0	0	0	0	0
1 vs. 3	Wide	32	31	40	9	34	4	32	10
	Narrow	0	0	0	0	0	0	0	0
1 vs. 4	Wide	39	32	38	28	40	38	32	24
	Narrow	8	0	0	0	0	0	0	0
1 vs. 5	Wide	39	37	39	26	40	32	34	26
	Narrow	3	0	0	0	9	0	0	0

*Error frequencies doubled.

a smooth reduction in size from the second to subsequent Eigenvalues in the unidimensional case when tetrachoric correlations can be reliably determined. As item difficulty distributions become more widespread, however, the sampling errors of tetrachorics also become more variable. Values associated with very easy and very difficult items are huge.

In addition to the substantial sensitivity to the distribution of item difficulties, this index shows increasing overlap with small _N_, small _n_, large factor intercorrelations, and large number of factors. With 60 items and the narrow range of item difficulties, overlap is zero even with five factors and high factor correlations.

Local Independence Index. Summary information appears in Table 5. This index is affected similarly by the five parameters, but at generally lower amounts of overlap, than the ratio of differences index. The local independence index is especially effective for large _N_, large _n_, narrow range of difficulties, and for the distinction between one and two factors.

Pattern of Second Factor Signs. Table 6 contains the overlap information for this index. Overall, errors in the detection of multidimensionality are more frequent for this index than for the ones based on ratios of Eigenvalue differences and on local independence, but the effect of one parameter is quite different. The two indices named were less effective in the wide than in the narrow range of item difficulties, but the pattern of second factor loadings reverses that effect. The latter index is at least as effective as the others in the wide range of item difficulties and presumably would become more effective in a still wider range.

Size of First and Second Factor Loadings. The information in Table 7 indicates that this index is only partially redundant with the pattern of

Table 5

Amount of Overlap in Each Cell

Based on the Local Independence Index

N=125

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	24	11	26	16	17	3	4	0
	Narrow	22	11	18	0	0	0	0	0
1 vs. 3	Wide	32	29	22	10	18	17	28	0
	Narrow	25	18	22	11	7	6	0	0
1 vs. 4	Wide	31	21	30	25	25	7	22	10
	Narrow	35	13	22	4	22	0	8	8
1 vs. 5	Wide	30	25	26	31	34	17	36	16
	Narrow	34	18	31	24	33	19	10	8

N=500

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	5	0	0	0	0	0	0	0
	Narrow	0	0	0	0	0	0	0	0
1 vs. 3	Wide	8	2	5	0	1	0	0	0
	Narrow	0	0	0	0	3	0	0	0
1 vs. 4	Wide	23	13	7	0	2	1	0	0
	Narrow	2	0	5	0	2	10	0	0
1 vs. 5	Wide	32	10	18	5	16	2	2	0
	Narrow	16	0	11	0	22	1	0	0

*Error frequencies doubled.

Table 6

Amount of Overlap in Each Cell

Based on Pattern of Second Factor Loadings

N=125

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	32	18	33	26	21	3	0	4
	Narrow	39	38	36	36	35	35	24	24
1 vs. 3	Wide	36	31	25	16	19	6	12	0
	Narrow	38	40	37	33	36	36	32	24
1 vs. 4	Wide	35	20	16	17	10	15	16	6
	Narrow	40	39	35	33	40	35	24	26
1 vs. 5	Wide	39	22	28	19	19	5	10	6
	Narrow	39	38	37	39	36	35	26	24

N=500

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	6	6	0	0	3	3	0	0
	Narrow	31	32	27	27	4	6	0	0
1 vs. 3	Wide	11	8	0	0	15	0	0	0
	Narrow	34	32	27	32	7	5	0	0
1 vs. 4	Wide	27	0	8	0	23	2	4	0
	Narrow	25	25	36	27	0	3	0	0
1 vs. 5	Wide	8	0	19	0	19	0	6	0
	Narrow	25	29	32	27	7	2	0	0

*Error frequencies doubled.

Table 7

Amount of Overlap on Each Cell

Based on Pattern of First and Second Factor Loadings

N=125

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	38	23	30	23	13	0	0	0
	Narrow	24	26	25	26	22	16	6	8
1 vs. 3	Wide	40	29	27	24	14	0	4	0
	Narrow	36	37	30	27	32	26	22	12
1 vs. 4	Wide	39	36	16	21	14	15	6	0
	Narrow	40	35	35	32	33	27	16	14
1 vs. 5	Wide	40	34	30	14	15	5	6	0
	Narrow	40	38	36	37	32	34	28	22

N=500

		20		30		40		60*	
		High	Low	High	Low	High	Low	High	Low
1 vs. 2	Wide	6	0	0	0	2	0	0	0
	Narrow	11	3	2	0	0	0	0	0
1 vs. 3	Wide	24	19	0	0	9	0	0	0
	Narrow	15	14	2	1	3	0	0	0
1 vs. 4	Wide	23	3	20	0	20	0	14	0
	Narrow	11	13	9	0	2	2	0	0
1 vs. 5	Wide	18	4	25	3	24	0	22	0
	Narrow	18	16	7	4	2	0	0	0

*Error frequencies doubled.

second factor signs. Overlap for the fifth index also tends to be greater in the narrow distribution of difficulties than in the wide, but the effect is smaller than for the fourth. For other combinations of parameters there is no substantial advantage for one or the other. In those cases where #5 does appear to be superior to #4, it tends to be inferior to #3. Overall, it is intermediate in its characteristics to the third and fourth indices and seems to have no unique advantage.

Main Effects of the Five Indices. Because we have a complete factorial design, it is possible to summarize the data for each index for each parameter by adding across all other parameters as in an analysis of variance. This will reveal more clearly the comparative strengths and weaknesses of the five indices. This information appears in Table 8.

Indices can first be compared by noting the sheer amount of overlap for each of the 15 main effects. Neglecting the fact that we do not, at the moment, have an error term, it is seen that the standardized difference in the size of the first two Eigenvalues does not discriminate best for any level of any parameter and has the largest total amount of overlap. The local independence index has the lowest total amount of overlap and is most effective for all *n*, *m*, *N*, and both levels of factor correlations. It is not at the top, however, for either the wide or narrow distributions of item difficulties. For the narrow distribution the ratio of differences in Eigenvalues is most effective and for the wide distribution the pattern of second factor signs is at the top. The latter index, however, is second only to the standardized difference in total number of errors.

It is also useful to compare proportions of errors of inference within a given parameter across the five indices. The two indices based on Eigenvalues are least sensitive to the increase in *N*, the ratio of differences

Table 8

Main Effects of Number and Proportion of Errors

Within Treatments for Each Index

	Standardized Difference		Ratio of Differences		Local Independence		Patterns of Loadings Factors 1 and 2		Factor 2	
Number of Items										
20	1025	.289	626	.311	490	.378	753	.418	843	.363
30	901	.254	533	.265	369	.285	506	.281	728	.313
40	897	.253	514	.255	285	.220	362	.201	485	.209
60	724	.204	340	.169	152	.117	180	.100	268	.115
Number of Factors										
1 vs. 2	771	.217	341	.169	157	.121	304	.169	549	.236
1 vs. 3	863	.243	440	.219	264	.204	447	.248	592	.255
1 vs. 4	931	.262	599	.298	348	.269	496	.275	587	.253
1 vs. 5	982	.277	633	.314	527	.407	554	.308	596	.256
Sample Size										
N=125	1992	.562	1128	.560	1072	.827	1430	.794	1654	.712
N=500	1555	.438	885	.440	224	.173	371	.206	670	.288
Difficulty Distribution										
Wide	2108	.594	1859	.924	795	.613	792	.440	733	.315
Narrow	1439	.466	154	.076	501	.387	1009	.560	1591	.685
Factor Correlations										
High	2012	.567	1205	.599	874	.674	1078	.599	1309	.563
Low	1535	.433	808	.401	422	.326	723	.401	1015	.437
Total Overlap All Treatments	3547		2013		1296		1801		2324	

is most sensitive by far to the distribution of item difficulties. The local independence index is most sensitive to an increase in the number of factors and comes close to losing its #1 position for five factors. The pattern of second factor signs stands out from the rest for the wide distribution of item difficulties and is relatively insensitive to the number of factors being compared.

All indices show the expected sensitivity to sample size, albeit somewhat differentially, to obliquity of the factors, and to the number of items. An investigator can control $\underline{N}$ and usually $\underline{n}$ as well without increasing the number of major factors in the items. Given the low level of item correlations found typically in cognitive tests, large samples and a very large ratio of number of variables to number of factors by ordinary factor analytic standards are important design considerations. The level of factor correlations, on the other hand, cannot be controlled by the investigator except indirectly and inversely with respect to the desired direction. As mentioned earlier, if a test is not unidimensional in spite of the item writer's best efforts, the multiple factors will probably be highly intercorrelated.

Discussion

This research has documented the complexity of the problem of dimensionality in binary items. No one index of the five we have tried works best in all combinations of our parameters. Only one index of the five we have tried can be rejected in toto: namely, the difference in size of Eigenvalues of the first two principal factors.

Development of a corpus of Monte Carlo runs allowing choice of an index of dimensionality that is contingent on knowable parameters will be difficult. The important parameters of factor intercorrelations and item difficulties were set at only two levels each, yet each varies continuously. Distributions

of item difficulties vary along more than one dimension as well, making generalization from several fixed distributions hazardous. *A priori* difficulty distributions can be approximated by skillful item writers, but factor intercorrelations cannot be kept at low levels by skilled test construction practices. Positive correlations among factors are intrinsic to the cognitive domain and are not trivial in size between group factors that are widely recognized as being truly different.

In spite of the difficulties we believe that useful recommendations can be developed. In a preliminary way we are offering to readers in Appendix A means and standard deviations of the three nontraditional methods used in the current research for each cell in the factorial design. Readers have a feel for the Eigenvalues of tetrachoric matrices, but lack this feel for the new indices. Readers should note that the skewness of the distributions limits the value of the standard deviations.

Because the new indices are quick and inexpensive makes further work on them attractive. On the basis of present information, full information factor analysis of Bock, Gibbons, and Muracki (1985) solves the problem of dimensionality in binary items, but it does so at a substantial cost. Computer time for this method increases rapidly as the number of items increases.

For the present we are working on several possible modifications of the indices that are dependent on raw score and standardized variance-covariance matrices. We have established that the attractive statistical properties of such matrices outweigh their dependency in binary items of percent passing.

References

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Drasgow, F. and Parsons, C. K. (1983) Application of unidimensional item response theory models to multidimensional data. Applied Psychological Measurement, 7, 189-199.

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Table A1

Means and Variances of the Local Independence Index

N=125

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
One Factor								
20	.57	.06	.57	.06	.58	.05	.58	.05
30	.61	.04	.61	.04	.64	.03	.64	.03
40	.63	.05	.63	.05	.64	.03	.64	.03
60	.67	.04	.67	.04	.70	.03	.70	.03
Two Factors								
20	.46	.09	.36	.09	.39	.11	.30	.14
30	.48	.10	.34	.10	.44	.12	.32	.12
40	.46	.09	.40	.10	.42	.08	.29	.13
60	.50	.08	.38	.07	.41	.05	.32	.12
Three Factors								
20	.51	.06	.44	.11	.48	.08	.41	.10
30	.52	.07	.44	.10	.51	.08	.42	.08
40	.53	.06	.48	.08	.55	.04	.43	.10
60	.57	.08	.46	.06	.54	.05	.44	.08

table continues

Table A1(cont.)

N=125

Four Factors

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
20	.52	.06	.45	.08	.55	.07	.42	.07
30	.54	.05	.49	.07	.54	.07	.47	.07
40	.56	.06	.50	.06	.56	.04	.49	.06
60	.62	.03	.56	.05	.58	.07	.52	.08

Five Factors

n	X	S_x	X	S_x	X	S_x	X	S_x
20	.50	.05	.44	.09	.55	.07	.44	.07
30	.54	.05	.51	.09	.57	.06	.51	.08
40	.60	.05	.53	.07	.59	.05	.53	.07
60	.65	.02	.56	.07	.62	.04	.56	.05

N=500

One Factor

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
20	.70	.04	.70	.04	.71	.03	.71	.03
30	.71	.03	.71	.03	.71	.02	.71	.02
40	.72	.02	.72	.02	.72	.03	.72	.03
60	.75	.02	.75	.02	.75	.01	.75	.01

table continues

Table A1 (cont.)

N=500

Two Factors

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
20	.45	.10	.33	.10	.29	.10	.18	.06
30	.36	.07	.28	.06	.24	.07	.14	.06
40	.33	.07	.26	.08	.23	.08	.13	.04
60	.32	.08	.20	.06	.21	.06	.13	.04
				Three Factors				
20	.54	.06	.45	.07	.47	.09	.41	.09
30	.52	.07	.44	.10	.43	.09	.36	.08
40	.51	.07	.39	.07	.43	.11	.36	.10
60	.50	.07	.43	.05	.46	.07	.40	.09
				Four Factors				
20	.60	.05	.55	.08	.52	.08	.46	.07
30	.57	.06	.48	.07	.54	.08	.48	.08
40	.58	.04	.51	.06	.52	.07	.44	.09
60	.58	.04	.48	.04	.51	.06	.47	.06
				Five Factors				
20	.61	.06	.57	.05	.59	.07	.54	.06
30	.59	.06	.52	.06	.60	.05	.54	.06
40	.63	.04	.55	.06	.59	.05	.51	.05
60	.63	.05	.56	.06	.58	.06	.52	.06

Table A2

Means and Variances of the Index Based on the Pattern of Signs of the Second Principal Factor

N=125

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_X	X	S_X	X	S_X	X	S_X
One Factor								
20	17	11	17	11	36	12	36	12
30	37	21	37	21	84	33	84	33
40	43	21	43	21	147	37	147	37
60	70	57	70	57	270	85	270	85
Two Factors								
20	30	14	41	7	39	7	39	7
30	68	29	88	24	100	14	96	14
40	133	46	154	25	174	26	170	28
60	360	66	357	69	400	41	362	61
Three Factors								
20	34	10	36	9	40	8	36	11
30	76	29	89	15	87	20	95	11
40	144	48	144	30	172	29	166	30
60	287	87	380	42	350	74	375	55

table continues

Table A2 (cont.)

N=125

Four Factors

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
20	34	12	40	11	39	10	42	9
30	90	18	86	15	91	13	98	12
40	128	35	164	38	155	26	169	21
60	211	66	316	75	400	35	394	34
				Five Factors				
20	30	10	38	11	38	11	39	8
30	73	31	86	19	92	18	96	20
40	139	42	143	34	171	29	170	29
60	267	109	316	103	371	49	403	47

N=500

One Factor

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
20	6	4	6	4	23	12	23	12
30	9	6	9	6	46	29	46	29
40	14	16	14	16	51	28	51	28
60	28	18	28	18	74	37	74	37

table continues

Table A2 (cont.)

N=500

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
Two Factors								
20	33	13	37	11	40	9	42	9
30	88	20	88	22	98	15	95	16
40	143	44	152	39	164	27	165	30
60	344	90	375	45	392	43	396	38
Three Factors								
20	25	13	30	12	39	10	41	13
30	74	28	90	21	101	15	92	19
40	105	58	157	31	156	29	171	29
60	313	120	367	39	384	47	389	43
Four Factors								
20	25	16	35	11	40	8	41	7
30	50	28	82	22	88	21	93	15
40	101	50	144	37	163	19	172	27
60	217	109	342	44	405	29	351	48
Five Factors								
20	31	13	40	8	39	7	38	7
30	48	35	76	23	93	18	94	15
40	78	50	155	25	154	28	177	26
60	124	91	333	92	390	43	377	49

Table A3

Means and Variances of the Index Based on the

Products of the First and Second Principal Factor Loadings

N=125

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
				One Factor				
20	.24	.16	.24	.16	.60	.18	.60	.18
30	.32	.15	.32	.15	.86	.38	.86	.38
40	.27	.12	.27	.12	1.07	.35	1.07	.44
60	.38	.13	.38	.13	1.38	.58	1.38	.58
				Two Factors				
20	.47	.24	.71	.23	.84	.21	.86	.31
30	.68	.36	.98	.35	1.32	.20	1.32	.34
40	1.15	.50	1.23	.25	1.77	.36	1.88	.44
60	1.87	.46	2.26	.38	2.98	.49	2.92	.60
				Three Factors				
20	.42	.19	.53	.18	.78	.21	.75	.29
30	.70	.37	.84	.29	1.09	.32	1.27	.32
40	1.08	.39	1.16	.24	1.45	.32	1.61	.39
60	1.30	.48	1.96	.47	2:21	.53	2.58	.40

table continues

Table A3 (cont.)

N=125

Four Factors

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
20	.47	.23	.61	.16	.66	.23	.76	.22
30	.77	.26	.77	.24	1.06	.28	1.12	.19
40	.88	.36	.98	.36	1.42	.29	1.54	.33
60	.90	.44	1.45	.36	2.42	.32	2.47	.32
				Five Factors				
20	.36	.19	.52	.20	.56	.19	.70	.19
30	.62	.30	.75	.22	1.01	.28	1.10	.27
40	.87	.37	.94	.30	1.41	.28	1.30	.29
60	1.14	.42	1.43	.50	1.96	.35	2.29	.29

N=500

One Factor

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_x	X	S_x	X	S_x	X	S_x
20	.07	.04	.07	.04	.24	.15	.24	.15
30	.08	.05	.08	.05	.25	.17	.25	.17
40	.10	.06	.10	.06	.24	.16	.24	.16
60	.16	.08	.16	.08	.29	.10	.29	.10

table continues

Table A3 (cont.)

N=500

	Wide				Narrow			
	High		Low		High		Low	
n	X	S_X	X	S_X	X	S_X	X	S_X
				Two Factors				
20	.46	.22	.57	.20	.78	.28	.88	.18
30	.82	.26	.91	.29	1.29	.29	1.40	.19
40	1.04	.45	1.22	.38	1.61	.25	1.79	.27
60	1.70	.48	2.33	.34	2.76	.42	3.02	.40
				Three Factors				
20	.32	.20	.47	.22	.62	.23	.68	.18
30	.58	.24	.75	.27	1.10	.17	1.07	.26
40	.67	.42	1.14	.24	1.27	.35	1.55	.29
60	1.36	.50	1.80	.22	2.01	.35	2.10	.36
				Four Factors				
20	.27	.19	.42	.16	.64	.16	.60	.19
30	.37	.24	.70	.22	.86	.25	1.13	.19
40	.61	.40	.94	.32	1.30	.24	1.38	.20
60	.70	.41	1.42	.33	1.84	.30	2.19	.25
				Five Factors				
20	.42	.17	.46	.13	.53	.14	.58	.16
30	.28	.25	.61	.24	.84	.18	.94	.19
40	.39	.28	.89	.27	1.06	.18	1.28	.19
60	.48	.46	1.40	.44	1.73	.25	2.00	.33

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